KDD-23 Research Track Paper

Shift-Robust Molecular Relational Learning with Causal Substructure

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BACKGROUND MOLECULAR RELATIONAL LEARNING



Molecular Relational Learning

Learning the interaction behavior between a pair of molecules

Examples

- Predicting solubility when a **drug** and **solvent** react
- Predicting side effects when taking two types of drugs simultaneously
- Predicting optical properties when a **Chromophore** and **Solvent** react

BACKGROUND DISTRIBUTION SHIFT IN MOLECULES

Core Substructure (NO2)



Molecule: 6-nitro-1H-indene



Molecular fingerprints with various scaffolds

Molecules with different scaffolds exhibit distinct distributions

- → Learning from core substructure is crucial for the robustness of machine learning (ML) models to distribution shifts
- \rightarrow Enabling ML models to learn more generalized knowledge in molecules!

* Molecules with nitrogen dioxide (NO2) functional group commonly exhibit the mutagenic property

* Scaffold: The common structure characterizing a group of molecules

BACKGROUND CAUSAL INFERENCE





Due to the empirical process of data collection, the data for machine learning is heavily biased

Context of the given data becomes a confounder that misleads the machine learning model to learn spurious correlations between pixels and labels

Ex) Spurious correlation between forest and lion in Figure

Causal Inference aims to improve model performance by removing spurious correlations

BACKGROUND CAUSAL INFERENCE FOR GRAPH STRUCTURED DATA



Determining House Motifs

Spurious correlation between the Tree motifs with House motifs

When facing with out-of-distribution (OOD) data, statistical shortcuts will severely deteriorates the model performance

BACKGROUND CAUSAL INFERENCE FOR GRAPH STRUCTURED DATA



4-iminocyclohexa-2,5-dien-1-one

(24) 4-acridine-9-yliminocyclohexa-2,5-dien-1-one 0 (0/1)

Instead of probing into the causal effect of the functional groups, Model focuses on "carbon rings" as the cues of the mutagenic class

In fact, "Carbon ring" has no relationship with mutagenicity

Spurious correlation becomes even severe in molecules!

BACKGROUND STRUCTURAL CAUSAL MODEL



Structure Causal Model (SCM) for molecular property prediction

Causal feature



Causal-Effect relationship in molecular property prediction

 $\underline{C} \leftarrow \underline{G} \rightarrow \underline{S}$: \underline{C} and \underline{S} naturally coexist in molecule \underline{G} .

<u> $C \rightarrow R \leftarrow S$ </u>: The variable <u>R</u> is the representation of the given molecule <u>G</u>.

- $\underline{C \rightarrow R \rightarrow Y}$: Causality we are interested in
- $\stackrel{\bullet}{\longrightarrow} \underline{C} \leftarrow \underline{G} \rightarrow \underline{S} \rightarrow \underline{R} \rightarrow \underline{Y}$: Backdoor path

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Shift-Robust Molecular Relational Learning with Causal Substructure

METHODOLOGY CAUSALITY IN MOLECULAR RELATIONAL LEARNING

Key causal-effect relationship in molecular relational learning



 G^1 : Molecule 1 G^2 : Molecule 2 C^1 : Causal Substructure in Molecule 1 S^1 : Shortcut Substructure in Molecule 1 \mathcal{R}^1 : Molecule 1 Representation \mathcal{R}^2 : Molecule 2 Representation Y: Target Value

Structure Causal Model (SCM) for Molecular Relational Learning



Causal substructure C^1 of molecule G^1 \rightarrow Determined by not only G^1 but also G^2



METHODOLOGY CAUSALITY IN MOLECULAR RELATIONAL LEARNING



 G^1 : Molecule 1 G^2 : Molecule 2 C^1 : Causal Substructure in Molecule 1 S^1 : Shortcut Substructure in Molecule 1 \mathcal{R}^1 : Molecule 1 Representation \mathcal{R}^2 : Molecule 2 Representation Y: Target Value

Structure Causal Model (SCM) for Molecular Relational Learning

Causality we are interested in $(\mathcal{C}^1 \rightarrow Y)$

4 Backdoor paths that confound the model

$$\begin{array}{l} \mathcal{C}^{1} \leftarrow \mathcal{G}^{1} \rightarrow \mathcal{S}^{1} \leftarrow \mathcal{G}^{2} \rightarrow \mathcal{R}^{2} \rightarrow Y \\ \mathcal{C}^{1} \leftarrow \mathcal{G}^{2} \rightarrow \mathcal{R}^{2} \rightarrow Y \\ \mathcal{C}^{1} \leftarrow \mathcal{G}^{2} \rightarrow \mathcal{S}^{1} \rightarrow \mathcal{R}^{1} \rightarrow Y \\ \mathcal{C}^{1} \leftarrow \mathcal{G}^{1} \rightarrow \mathcal{S}^{1} \rightarrow \mathcal{R}^{1} \rightarrow Y \end{array}$$

In molecular relational learning, G^2 is given and utilized during model prediction

 $\mathcal{C}^1 \leftarrow \mathcal{G}^1 \rightarrow \mathcal{S}^1 \rightarrow \mathcal{R}^1 \rightarrow Y$ Only remaining backdoor path!

METHODOLOGY BACKDOOR ADJUSTMENT



 \mathcal{G}^1 : Molecule 1 \mathcal{G}^2 : Molecule 2 \mathcal{C}^1 : Causal Substructure in Molecule 1 \mathcal{S}^1 : Shortcut Substructure in Molecule 1 \mathcal{R}^1 : Molecule 1 Representation \mathcal{R}^2 : Molecule 2 Representation Y: Target Value

Structure Causal Model (SCM) for Molecular Relational Learning $P(\mathbf{Y}|do(C^{1}), \mathcal{G}^{2}) = \tilde{P}(\mathbf{Y}|C^{1}, \mathcal{G}^{2})$ = $\sum_{s} \tilde{P}(\mathbf{Y}|C^{1}, \mathcal{G}^{2}, s) \cdot \tilde{P}(s|C^{1}, \mathcal{G}^{2})$ (Bayes' Rule) = $\sum_{s} \tilde{P}(\mathbf{Y}|C^{1}, \mathcal{G}^{2}, s) \cdot \tilde{P}(s|\mathcal{G}^{2})$ (Independence) = $\sum_{s} P(\mathbf{Y}|C^{1}, \mathcal{G}^{2}, s) \cdot P(s|\mathcal{G}^{2}),$

Backdoor Adjustment

Alleviate confounding effect via Backdoor adjustment!



Disentangling with Atom Representation Masks

Separate the causal substructure C^1 and shortcut substructure S^1 from $G^1 \rightarrow Not$ trivial to explicitly manipulate molecular structure

 \rightarrow Let's separate in representation space by masking atom representation!

 $p_{i} = MLP(\mathbf{H}_{i}^{1}) \qquad \text{Importance of atom } i$ $C_{i}^{1} = \lambda_{i}\mathbf{H}_{i}^{1} + (1 - \lambda_{i})\epsilon \qquad \text{Causal substructure} \qquad \text{where} \\ S_{i}^{1} = (1 - \lambda_{i})\mathbf{H}_{i}^{1} \qquad \text{Shortcut substructure} \qquad \text{Where} \\ \lambda_{i} \sim \text{Bernoulli}(p_{i}) \quad \epsilon \sim N(\mu_{\mathrm{H}^{1}}, \sigma_{\mathrm{H}^{1}}^{2})$

Gumbel sigmoid approach for differentiable optimization of p_i



Disentangling with Atom Representation Masks

Causal substructure \mathcal{C}^1

- → Cross entropy loss for classification $\implies \mathcal{L}_{causal}(\mathbf{Y}, z_{C^1}, z_{G^2})$
- \rightarrow RMSE loss for Regression

Shortcut substructure S^1 \rightarrow Learn non informative distribution 

Regression Classification





Backdoor Adjustment



Conditional Causal Intervention via backdoor adjustment

Straightforward approach \rightarrow Synthesize / Collect various molecules

Challenges

1) Expensive time/financial costs

2) Intervention space on \mathcal{C}^1 should be conditioned on the paired molecule \mathcal{G}^2

Our Solution

Obtain shortcut substructure \tilde{S}^1 by modeling interaction with other molecules \tilde{G}^1 and molecule G^2

$$\mathcal{L}_{int} = \sum_{(\mathcal{G}^1, \mathcal{G}^2) \in \mathcal{D}} \sum_{\tilde{\mathcal{S}}^1} \mathcal{L}(\mathbf{Y}, z_{\mathcal{C}^1}, z_{\mathcal{G}^2}, z_{\tilde{\mathcal{S}}^1})$$



Final Objective

$$\mathcal{L}_{final} = \mathcal{L}_{sup} + \mathcal{L}_{causal} + \lambda_1 \cdot \mathcal{L}_{KL} + \lambda_2 \cdot \mathcal{L}_{int}$$

 \mathcal{L}_{sup} : loss with paired graph ($\mathcal{G}^1, \mathcal{G}^2$) and target Y \mathcal{L}_{causal} : loss with causal substructure \mathcal{L}_{KL} : loss with shortcut substructure λ_1, λ_2 : weight hyperparameters for \mathcal{L}_{KL} and \mathcal{L}_{int}

EXPERIMENTS DATASET DESCRIPTION

D	Dataset	\mathcal{G}^1	\mathcal{G}^2	$\# \mathcal{G}^1$	$\# \mathcal{G}^2$	# Pairs	Task
Chro-	Absorption	Chrom.	Solvent	6416	725	17276	MI
moph-	Emission	Chrom.	Solvent	6412	1021	18141	MI
ore ³	Lifetime	Chrom.	Solvent	2755	247	6960	MI
М	NSol ⁴	Solute	Solvent	372	86	86 2275	
Fre	eeSolv ⁵	Solute	Solvent	560	1	560	MI
Co	mpSol ⁶	Solute	Solvent	442	259	3548	MI
Abraham ⁷		Solute	Solvent	1038	122	6091	MI
Con	nbiSolv ⁸	Solute	Solvent	1495	326	10145	MI
ZhangDDI ⁹		Drug	Drug	544	544	40255	DDI
ChChMiner ¹⁰		Drug	Drug	949	949	21082	DDI
DeepDDI ¹¹		Drug	Drug	1704	1704	191511	DDI
AIDS ¹²		Mole.	Mole.	700	700	490K	SL
LINUX ¹²		Program	Program	1000	1000	1M	SL
IN	IMDB ¹²		Ego-net.	1500	1500	2.25M	SL
OpenSSL ¹³		Flow	Flow	4308 4308		18.5M	SL
FF1	mpeg ¹³	Flow	Flow	10824	10824	117M	SL

Molecular Interaction Dataset

- \rightarrow Predicting Chromophores' Absorption max, Emission max, Lifetime
- → Predicting Solvation Free Energy of molecules (MNSol, FreeSolv, CompSol, Abraham, CombiSolv)
- \rightarrow Regression Task

Drug-Drug Interaction Dataset

- → Zhang DDI, ChChMiner, DeepDDI
- \rightarrow Classification Task

Graph Similarity Learning Dataset

- \rightarrow How similar are the paired graphs? (ex. GED)
- → AIDS, LINUX, IMDB, OpenSSL, Ffmpeg
- → Regression Task / Classification Task

EXPERIMENTS OVERALL PERFORMANCE

	Chromophore			MNSol	FreeSolv	CompSol	Abraham	CombiSoly	
	Absorption	Emission	Lifetime	WINSOI	Treeson	compoor	Abraham	Combisorv	
GCN	25.75 (1.48)	31.87 (1.70)	0.866 (0.015)	0.675 (0.021)	1.192 (0.042)	0.389 (0.009)	0.738 (0.041)	0.672 (0.022)	
GAT	26.19 (1.44)	30.90 (1.01)	0.859 (0.016)	0.731 (0.007)	1.280 (0.049)	0.387 (0.010)	0.798 (0.038)	0.662 (0.021)	
MPNN	24.43 (1.55)	30.17 (0.99)	0.802 (0.024)	0.682 (0.017)	1.159 (0.032)	0.359 (0.011)	0.601 (0.035)	0.568 (0.005)	
GIN	24.92 (1.67)	32.31 (0.26)	0.829 (0.027)	0.669 (0.017)	1.015 (0.041)	0.331 (0.016)	0.648 (0.024)	0.595 (0.014)	
CIGIN	19.32 (0.35)	25.09 (0.32)	0.804 (0.010)	0.607 (0.024)	0.905 (0.014)	0.308 (0.018)	0.411 (0.008)	0.451 (0.009)	
CMRL	17.93 (0.31)	24.30 (0.22)	0.776 (0.007)	0.551 (0.017)	0.815 (0.046)	0.255 (0.011)	0.374 (0.011)	0.421 (0.008)	

Performance on molecular interaction prediction task

	AIDS			LINUX				IMDB		FFmpeg	OpenSSL
	MSE	ho	p@10	MSE	ρ	p@10	MSE	ho	p@10	AUROC	AUROC
SimGNN	1.376	0.824	0.400	2.479	0.912	0.635	1.264	0.878	0.759	93.45	94.25
GMN	4.610	0.672	0.200	2.571	0.906	0.888	4.422	0.725	0.604	94.76	93.91
GraphSim	1.919	0.849	0.446	0.471	0.976	0.956	0.743	0.926	0.828	94.48	93.66
HGMN	1.169	0.905	0.456	0.439	0.985	0.955	0.335	0.919	0.837	97.83	95.87
H ² MN _{RW}	0.936	0.878	0.496	0.136	0.988	0.970	0.296	0.918	0.872	99.05	92.21
H^2MN_{NE}	0.924	0.883	0.511	0.130	0.990	0.978	0.297	0.889	0.875	98.16	98.25
CMRL	0.770	0.899	0.574	0.094	0.992	0.989	0.263	0.944	0.879	98.69	96.57

Performance on graph similarity learning task

Observations

- 1. CMRL outperforms all other baseline methods
- ightarrow It is crucial to discover causally related substructure in molecules
- 2. Wide applicability of CMRL beyond molecules
- ightarrow Performs well in dataset that contains core substructure

EXPERIMENTS OUT-OF-DISTRIBUTION PERFORMANCE

In out-of-distribution experiment, we assess the model's performance on molecules belonging to new scaffold classes



Different scaffolds exhibit totally different distribution

TSNE on splitted data (Train / Test)

EXPERIMENTS OUT-OF-DISTRIBUTION PERFORMANCE

In out-of-distribution experiment, we assess the model's performance on molecules belonging to new scaffold classes

	(a) In-Distribution							(b) Out-of-Distribution					
	ZhangDDI		ChChMiner		DeepDDI		ZhangDDI		ChChMiner		DeepDDI		
	AUROC	Accuracy	AUROC	Accuracy	AUROC	Accuracy	AUROC	Accuracy	AUROC	Accuracy	AUROC	Accuracy	
GCN	91.64 (0.31)	83.31 (0.61)	94.71 (0.33)	87.36 (0.24)	92.02 (0.01)	86.96 (0.02)	70.61 (2.32)	64.22 (1.64)	74.17 (0.89)	67.56 (1.29)	76.38 (0.43)	67.92 (0.81)	
GAT	92.10 (0.28)	84.14 (0.38)	96.15 (0.53)	89.49 (0.88)	92.01 (0.02)	86.99 (0.05)	73.15 (2.50)	65.14 (2.47)	75.64 (0.99)	68.61 (0.72)	76.44 (1.27)	67.94 (1.38)	
MPNN	92.34 (0.35)	84.56 (0.31)	96.25 (0.53)	90.02 (0.42)	92.02 (0.02)	86.97 (0.01)	72.39 (1.70)	64.55 (1.75)	76.40 (0.91)	68.51 (0.71)	79.03 (0.81)	71.23 (0.90)	
GIN	93.16 (0.04)	85.59 (0.05)	97.52 (0.05)	91.89 (0.66)	92.03 (0.00)	87.02 (0.03)	75.04 (0.63)	67.14 (1.03)	74.32 (2.93)	67.49 (2.44)	78.61 (0.58)	70.33 (1.11)	
MIRACLE	93.05 (0.07)	84.90 (0.36)	88.66 (0.37)	84.29 (0.14)	62.23 (0.75)	62.35 (0.30)	59.57 (0.90)	52.31 (2.24)	73.28 (0.71)	50.49 (0.59)	62.32 (1.63)	51.30 (0.29)	
SSI-DDI	92.74 (0.12)	84.61 (0.18)	98.44 (0.08)	93.50 (0.16)	93.97 (0.38)	88.44 (0.39)	71.67 (4.71)	65.78 (3.02)	75.59 (1.93)	68.75 (1.41)	80.41 (1.74)	72.05 (1.47)	
CIGIN	93.28 (0.13)	85.54 (0.30)	98.51 (0.10)	93.77 (0.25)	99.12 (0.03)	96.55 (0.11)	73.99 (1.74)	66.44 (1.07)	80.24 (2.00)	73.28 (1.08)	83.78 (0.87)	74.07 (1.19)	
CMRL	93.73 (0.15)	86.32 (0.23)	98.70 (0.05)	94.26 (0.28)	99.13 (0.02)	96.70 (0.12)	75.30 (1.39)	67.76 (1.41)	82.05 (0.67)	74.21 (0.78)	83.83 (0.97)	75.20 (0.66)	

Performance on drug-drug interaction task

Observation

CMRL outperforms previous work on out-of-distribution scenarios

 \rightarrow Learning causal substructure enhances the generalization ability of the model

EXPERIMENTS MODEL ANALYSIS



Observations in Ablation Studies

Naïve intervention whose confounders are not conditioned on paired molecule G^2 \rightarrow Performs worse than the model without intervention

 \rightarrow Wideness of intervention space introduces noisy signal during model training



Observations in Sensitivity Analysis

- 1. Optimal point for λ_2 exist balancing the noisiness and robustness
- 2. No certain relationship between model performance and λ_1

Training objective

 $\mathcal{L}_{final} = \mathcal{L}_{sup} + \mathcal{L}_{causal} + \lambda_1 \cdot \mathcal{L}_{KL} + \lambda_2 \cdot \mathcal{L}_{int}$

EXPERIMENTS QUALITATIVE ANALYSIS



Observations

- 1. Discovered causal substructure aligns to well-known chemical domain knowledge
- (a) CMRL selects edge substructure \rightarrow Chemical reactions usually happen around ionized atoms
- (b) CMRL concentrates on single-bonded substructure \rightarrow Single-bonded substructures are more likely to undergo chemical reactions
- 2. (c) When reacting with polar solvents, CMRL focuses on the edge substructures of high polarity
- 3. (d) Selected important substructures of chromophore varies as the solvent varies

CONCLUSION

This paper proposed a method for tackling relation learning tasks, which are prevalent in various scientific field

Keyword: Conditional causal intervention

 \rightarrow Crucial to narrow down intervention space by conditioning on paired molecule \mathcal{G}^2

Extensive experiments demonstrating the superiority and interpretability of CMRL \rightarrow Makes CMRL highly practical for real-world scientific discovery

[Full Paper] https://arxiv.org/abs/2305.18451

[Source Code] https://github.com/Namkyeong/CMRL

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Appendix

<u>Task: Rationalization for GNNs</u> \rightarrow "What knowledge drives the GNNs to make certain predictions?"

Invariant Learning

ightarrow Constructs different environments to infer the invariant features or predictors



Generate s-interventional distribution by doing intervention on \boldsymbol{S}

Definition 1 (DIR Principle) An intrinsically-interpretable model h satisfies the DIR principle if it

1. minimizes all s-interventional risks: $\mathbb{E}_{s}[\mathcal{R}(h(G), Y | do(S = s))]$, and simultaneously

2. minimizes the variance of various s-interventional risks: $Var_s(\{\mathcal{R}(h(G), Y | do(S = s))\})$, where the s-interventional risk is defined over the s-interventional distribution for specific $s \in S$.

 $\min \mathcal{R}_{\text{DIR}} = \mathbb{E}_s[\mathcal{R}(h(G), Y | do(S = s))] + \lambda \text{Var}_s(\{\mathcal{R}(h(G), Y | do(S = s))\})$

1. Minimize the risk under all *s*-interventional distributions

2. Minimize variance of risk over different *s*-interventional distributions



Model Architecture

<u>Rationale Generator</u> Split the input graph instance $g = (\mathcal{V}, \mathcal{E})$ into two subgraphs: causal part \tilde{c} and non-causal part \tilde{s}

Distribution Intervener

Collects non-causal part of all instances into a memory bank as \tilde{S} Samples memory $\tilde{s}_i \in \tilde{S}$ to conduct intervention $do(S = \tilde{s}_i)$, constructing an intervened pair $(\tilde{c}_i, \tilde{s}_i)$

Model Prediction

$$\hat{y} = \hat{y}_{\tilde{c}} \odot \sigma(\hat{y}_{\tilde{s}})$$

Optimization

$$\mathcal{R}(h(G), Y | do(S = \tilde{s})) = \mathbb{E}_{(g,y) \in \mathcal{O}, S = \tilde{s}, C = h_{\tilde{C}}(g)} l(\hat{y}, y)$$
$$\mathcal{R}_{\tilde{S}} = \mathbb{E}_{(g,y) \in \mathcal{O}, \tilde{s} = g/h_{\tilde{C}}(g)} l(\hat{y}_{\tilde{s}}, y)$$

<u>Task: Graph Classification</u> \rightarrow "How to classify biased graph datasets?"



Model Architecture

Soft Mask Estimation

Separate the causal and shortcut features from the full graphs

Disentanglement

Causal graph

Trivial graph

Separate the causal and shortcut features from the full graphs

$$\mathbf{h}_{\mathcal{G}_c} = f_{\text{readout}}(\text{GConv}_c(\mathbf{A} \odot \mathbf{M}_a, \mathbf{X} \odot \mathbf{M}_x)), \quad \mathbf{z}_{\mathcal{G}_c} = \Phi_c(\mathbf{h}_{\mathcal{G}_c})$$
$$\mathbf{h}_{\mathcal{G}_t} = f_{\text{readout}}(\text{GConv}_t(\mathbf{A} \odot \overline{\mathbf{M}}_a, \mathbf{X} \odot \overline{\mathbf{M}}_x)), \quad \mathbf{z}_{\mathcal{G}_t} = \Phi_t(\mathbf{h}_{\mathcal{G}_t})$$

 $\mathcal{L}_{sup} = -\frac{1}{|\mathcal{D}|} \sum_{\mathcal{G} \in \mathcal{D}} \mathbf{y}_{\mathcal{G}}^{\top} \log(\mathbf{z}_{\mathcal{G}_c}) \qquad \text{Causal graph} \rightarrow \text{Ground truth label prediction}$

$$\mathcal{L}_{\text{unif}} = \frac{1}{|\mathcal{D}|} \sum_{\mathcal{G} \in \mathcal{D}} \text{KL}(\mathbf{y}_{\text{unif}}, \mathbf{z}_{\mathcal{G}_t})$$

Trivial graph \rightarrow Random label prediction



Structure Causal Model (SCM)

$$P(Y|do(C)) = P_m(Y|C)$$

= $\sum_{s \in \mathcal{T}} P_m(Y|C, s) P_m(s|C)$ (Bayes Rule)
= $\sum_{s \in \mathcal{T}} P_m(Y|C, s) P_m(s)$ (Independency)
= $\sum_{s \in \mathcal{T}} P(Y|C, s) P(s)$, Confounder Set

Backdoor Adjustment

Causal Intervention via Backdoor adjustment

Challenges

Confounder set *T* is commonly unobservable and hard to obtain
Difficult to directly manipulate graph data (∵Discrete nature)

Let's make implicit intervention on representation level!

 $\mathbf{z}_{\mathcal{G}'} = \Phi(\mathbf{h}_{\mathcal{G}_c} + \mathbf{h}_{\mathcal{G}_{t'}})$ Trivial graph from different graphs

$$\mathcal{L}_{\text{caus}} = -\frac{1}{|\mathcal{D}| \cdot |\hat{\mathcal{T}}|} \sum_{\mathcal{G} \in \mathcal{D}} \sum_{t' \in \hat{\mathcal{T}}} \mathbf{y}_{\mathcal{G}}^{\top} \log \left(\mathbf{z}_{\mathcal{G}'} \right)$$

<u>Task: Graph Classification</u> \rightarrow "How to classify biased graph datasets?"



Model Architecture

<u>Causal and Bias Substructure Generator</u> Measure the edge importance between node v_i and v_j

$$\alpha_{ij} = \text{MLP}([\mathbf{x}_i, \mathbf{x}_j]) \xrightarrow{\text{Edge in}} c_{ij} = \sigma(\alpha_{ij})$$

Learning Disentangled Graph Representations

Bias GNN \rightarrow Generalized cross entropy loss Causal GNN \rightarrow Weighted cross entropy loss



Model Architecture

Counterfactual Unbiased Sample Generation

How to make causal variable z_c and bias variable z_b uncorrelated? Swapping z_b with randomly selected different graphs

$$z_{unbiased} = [z_c; \hat{z_b}] \quad \text{From different graphs}$$
$$L_G = W(z)CE(C_c(z_{unbiased}), y) + GCE(C_b(z_{unbiased}), \hat{y})$$

Can be considered as Backdoor adjustment!

BACKGROUND CAUSAL INFERENCE FOR GRAPH STRUCTURED DATA

Causal view of data-generating process

Input graph G consists of two disjoint part:
Causal part C and Non-causal part S $C \rightarrow G \leftarrow S$ Create spurious correlation between S and Y $C \leftarrow \cdots \leftarrow S$ Structure Causal Model
(SCM)Causal part C only determines target value Y $C \leftarrow \cdots \leftarrow Y$

THEORETICAL ANALYSIS

Training objective of CMRL $-\ell = -\sum_{i=1}^{n} \log q(\mathbf{Y}_i | C_i^1, \mathcal{G}_i^2)$

Expand by multiplying and dividing q

$$\begin{split} -\ell &= \sum_{i=1}^{n} \log \frac{p(\mathbf{Y}_{i} | C_{i}^{1}, \mathcal{G}_{i}^{2})}{q(\mathbf{Y}_{i} | C_{i}^{1}, \mathcal{G}_{i}^{2})} + \sum_{i=1}^{n} \log \frac{p(\mathbf{Y}_{i} | \mathcal{G}_{i}^{1}, \mathcal{G}_{i}^{2})}{p(\mathbf{Y}_{i} | C_{i}^{1}, \mathcal{G}_{i}^{2})} - \sum_{i=1}^{n} \log p(\mathbf{Y}_{i} | \mathcal{G}_{i}^{1}, \mathcal{G}_{i}^{2}) \\ &= \mathbb{E} \left[\log \frac{p(\mathbf{Y} | \mathcal{C}^{1}, \mathcal{G}^{2})}{q(\mathbf{Y} | \mathcal{C}^{1}, \mathcal{G}^{2})} \right] + \mathbb{E} \left[\log \frac{p(\mathbf{Y} | \mathcal{G}^{1}, \mathcal{G}^{2})}{p(\mathbf{Y} | \mathcal{C}^{1}, \mathcal{G}^{2})} \right] - \mathbb{E} \left[\log p(\mathbf{Y} | \mathcal{G}^{1}, \mathcal{G}^{2}) \right], \\ \mathbb{E} \left[\log \frac{p(\mathbf{Y} | \mathcal{G}_{i}^{1}, \mathcal{G}_{i}^{2})}{p(\mathbf{Y} | \mathcal{C}_{i}^{1}, \mathcal{G}_{i}^{2})} \right] = \mathbb{E} \left[\log \frac{p(\mathbf{Y} | \mathcal{C}_{i}^{1}, \mathcal{S}_{i}^{1}, \mathcal{G}_{i}^{2})}{p(\mathbf{Y} | \mathcal{C}_{i}^{1}, \mathcal{G}_{i}^{2})} \right] \\ &= \sum_{i=1}^{n} p(\mathcal{G}_{i}^{1}, \mathcal{G}_{i}^{2}, \mathbf{Y}_{i}) \log \frac{p(\mathbf{Y}_{i} | \mathcal{C}_{i}^{1}, \mathcal{S}_{i}^{1}, \mathcal{G}_{i}^{2})}{p(\mathbf{Y}_{i} | \mathcal{C}_{i}^{1}, \mathcal{G}_{i}^{2})} \\ &= \sum_{i=1}^{n} p(\mathcal{G}_{i}^{1}, \mathcal{G}_{i}^{2}, \mathbf{Y}_{i}) \log \frac{p(\mathbf{Y}_{i} | \mathcal{C}_{i}^{1}, \mathcal{G}_{i}^{2})}{p(\mathbf{Y}_{i} | \mathcal{C}_{i}^{1}, \mathcal{G}_{i}^{2})} \\ &= \sum_{i=1}^{n} p(\mathcal{G}_{i}^{1}, \mathcal{G}_{i}^{2}, \mathbf{Y}_{i}) \log \frac{p(\mathbf{Y}_{i} | \mathcal{C}_{i}^{1}, \mathcal{G}_{i}^{2})}{p(\mathbf{Y}_{i} | \mathcal{C}_{i}^{1}, \mathcal{G}_{i}^{2})} \\ &= \sum_{i=1}^{n} p(\mathcal{G}_{i}^{1}, \mathcal{G}_{i}^{2}, \mathbf{Y}_{i}) \log \frac{p(\mathcal{S}_{i}^{1}, \mathbf{Y}_{i} | \mathcal{C}_{i}^{1}, \mathcal{G}_{i}^{2})}{p(\mathbf{Y}_{i} | \mathcal{C}_{i}^{1}, \mathcal{G}_{i}^{2})} \\ &= \sum_{i=1}^{n} p(\mathcal{G}_{i}^{1}, \mathcal{G}_{i}^{2}, \mathbf{Y}_{i}) \log \frac{p(\mathcal{S}_{i}^{1}, \mathbf{Y}_{i} | \mathcal{C}_{i}^{1}, \mathcal{G}_{i}^{2})}{p(\mathcal{Y}_{i} | \mathcal{C}_{i}^{1}, \mathcal{G}_{i}^{2})} \\ &= I(\mathcal{S}^{1}; \mathbf{Y} | \mathcal{C}^{1}, \mathcal{G}^{2}) \end{split}$$

$$\min \mathbb{E}\left[\log \frac{p(\mathbf{Y}|\mathcal{C}^1, \mathcal{G}^2)}{q(\mathbf{Y}|\mathcal{C}^1, \mathcal{G}^2)}\right] + \frac{I(\mathcal{S}^1; \mathbf{Y}|\mathcal{C}^1, \mathcal{G}^2)}{H(\mathbf{Y}|\mathcal{G}^1, \mathcal{G}^2)} + H(\mathbf{Y}|\mathcal{G}^1, \mathcal{G}^2)$$

Likelihood ratio between true distribution and predicted distribution
Conditional Mutual Information
Irreducible constant inherent in the datasets

We can explain the behavior of CMRL in two perspective

THEORETICAL ANALYSIS

 $\min \mathbb{E}\left[\log \frac{p(\mathbf{Y}|\mathcal{C}^1, \mathcal{G}^2)}{q(\mathbf{Y}|\mathcal{C}^1, \mathcal{G}^2)}\right] + I(\mathcal{S}^1; \mathbf{Y}|\mathcal{C}^1, \mathcal{G}^2) + H(\mathbf{Y}|\mathcal{G}^1, \mathcal{G}^2)$

Perspective 1. CMRL learns informative causal substructure

Minimize $I(S^1; Y | C^1, G^2)$ Disentangle the shortcut substructure S^1 that are no longer needed in predicting the label Y when the context C^1 and G^2 given.

Chain rule of MI $I(\mathcal{S}^1; \mathbf{Y} | C^1, \mathcal{G}^2) = I(\mathcal{G}^1, \mathcal{G}^2; \mathbf{Y}) - I(C^1, \mathcal{G}^2; \mathbf{Y})$

Encourages the causal substructure C^1 and paired molecule G^2 to contain enough information on target Y.

THEORETICAL ANALYSIS

$$\min \mathbb{E}\left[\log \frac{p(\mathbf{Y}|\mathcal{C}^1, \mathcal{G}^2)}{q(\mathbf{Y}|\mathcal{C}^1, \mathcal{G}^2)}\right] + \frac{I(\mathcal{S}^1; \mathbf{Y}|\mathcal{C}^1, \mathcal{G}^2)}{H(\mathbf{Y}|\mathcal{G}^1, \mathcal{G}^2)} + H(\mathbf{Y}|\mathcal{G}^1, \mathcal{G}^2)$$

Perspective 2. CMRL reduces model bias with causal view



 G^1 : Molecule 1 G^2 : Molecule 2 C^1 : Causal Substructure in Molecule 1

 \mathcal{S}^{1} : Shortcut Substructure in Molecule 1

- \mathcal{R}^1 : Molecule 1 Representation
- \mathcal{R}^2 : Molecule 2 Representation
- Y : Target Value

Based on information leakage, Model bias can be quantified based on mutual information

Again, several backdoor paths are blocked by conditioning on \mathcal{C}^1 and \mathcal{G}^2

- \rightarrow Enable the direct measure of model bias!
- ightarrow Finally, Loss term minimize the model bias



Model bias

EXPERIMENTS SYNTHETIC DATASET EXPERIMENTS

In synthetic dataset experiment, we assess the model's performance on various levels of bias in datasets



Positive pair

a pair that shares the same causal substructure $\{\text{House}, \text{House}\} \rightarrow \text{Positive}$

Negative pair

a pair that each graph has a different causal substructure $\{\text{House, Cycle}\} \rightarrow \text{Negative}$

Dataset bias

the ratio of the positive pairs containing "BA." shortcut substructures

 $bias(b) = \frac{\text{Number of positive pairs with BA substructure}}{\text{Number of positive pairs}}$ $= \frac{\#\{\text{Causal-BA}, \text{Causal-BA}\}}{\#\{\text{Causal-Tree}\} + \#\{\text{Causal-BA}, \text{Causal-BA}\}}$

Bias level *b* increases

 \rightarrow "BA." substructures dominates model prediction

EXPERIMENTS SYNTHETIC DATASET EXPERIMENTS

In synthetic dataset experiment, we assess the model's performance on various levels of bias in datasets



Observations

- 1. Models' performance degrades as the bias gets severe
- \rightarrow "BA." shortcut confound the model

2. Performance gap between CMRL and CIGIN gets larger as the bias gets severe → Importance of learning causality between the substructure and target